Find the limit algebraically, verify w/ graphing calc.

① $\lim\limits_{x \to 0} \dfrac{x}{\sqrt{9+x} - 3}$

$$\frac{x}{\sqrt{9+x} - 3}\left(\frac{\sqrt{9+x}+3}{\sqrt{9+x}+3}\right) \longrightarrow \frac{x\left(\sqrt{9+x}+3\right)}{9+x-9}$$

$$\frac{\cancel{x}\left(\sqrt{9+x}+3\right)}{\cancel{x}} \longrightarrow \sqrt{9+x}+3 \longrightarrow \sqrt{9+0}+3 = 6$$

② $\lim\limits_{x \to 0} \dfrac{\sin 3x}{\sin 5x}$

③ $\lim\limits_{x \to 0} \dfrac{x^2 - x - 2}{x - 2}$

$$\frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}} = (x+1)$$

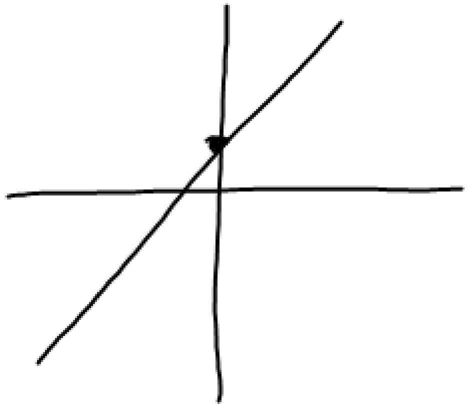
$$(0+1) = \boxed{1}$$

④ $\lim\limits_{x \to 2} \dfrac{x^2 - x - 2}{x - 2}$

$$(2+1)$$

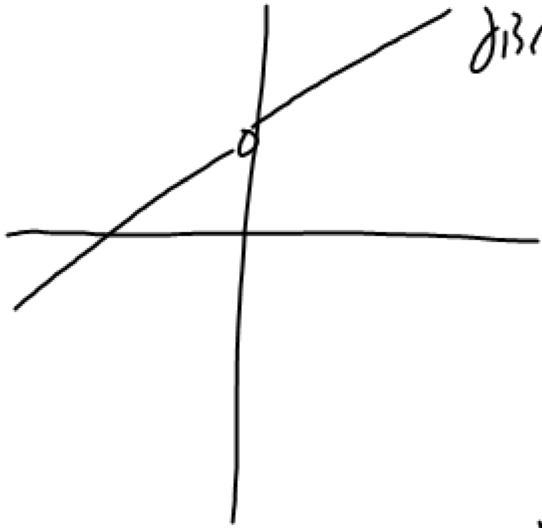
$$3$$

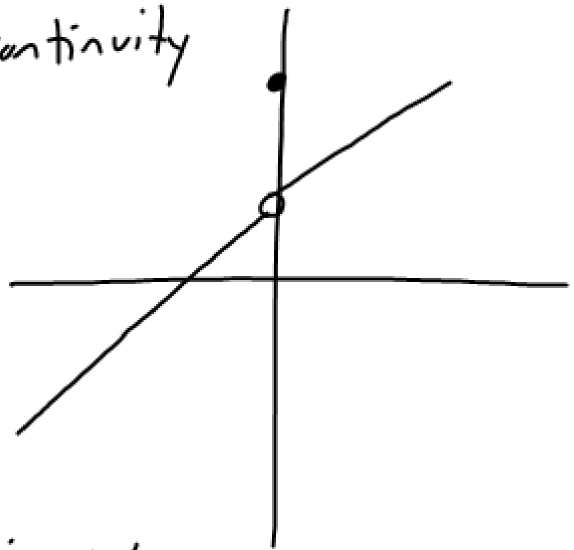
Continuity



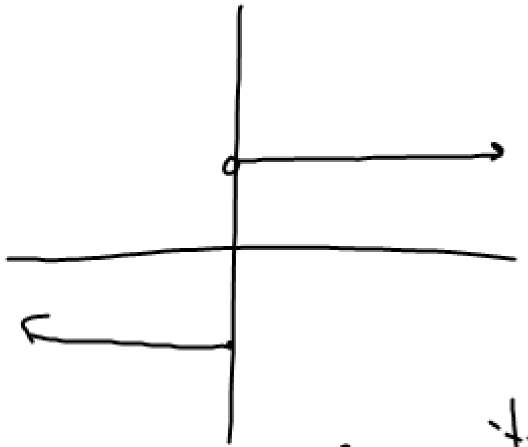
Continuous

Removable discontinuity

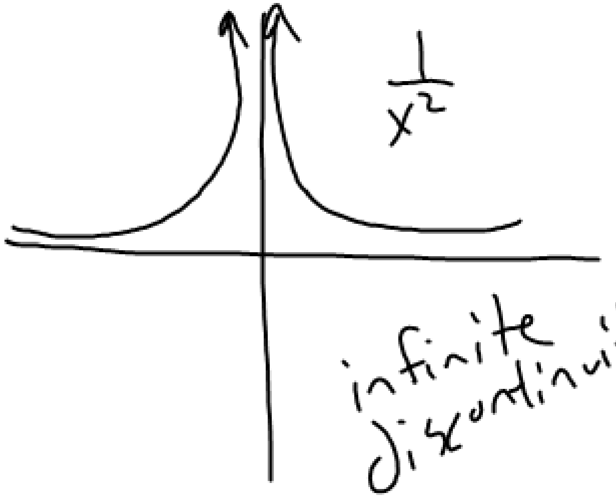




Non-continuous



jump discontinuity



$\frac{1}{x^2}$

infinite discontinuity

$\sin \frac{1}{x}$

oscillating discontinuity

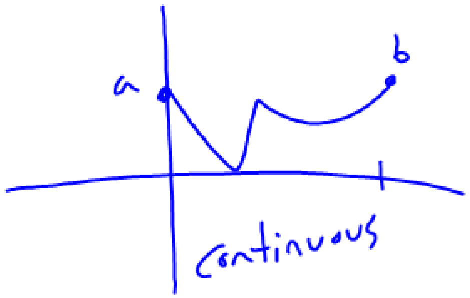
Continuity at point

Interior → A function $f(x)$ is continuous at an interior pt. c of its domain if

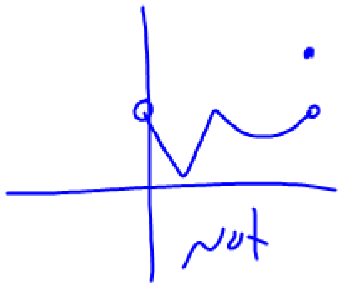
$$\lim_{x\to c} f(x) = f(c)$$

End point → A function $f(x)$ is continuous at a left endpoint a or a right endpoint b if

$$\lim_{x\to a^+} f(x) = f(a) \quad \text{or} \quad \lim_{x\to b^-} f(x) = f(b)$$



vs.

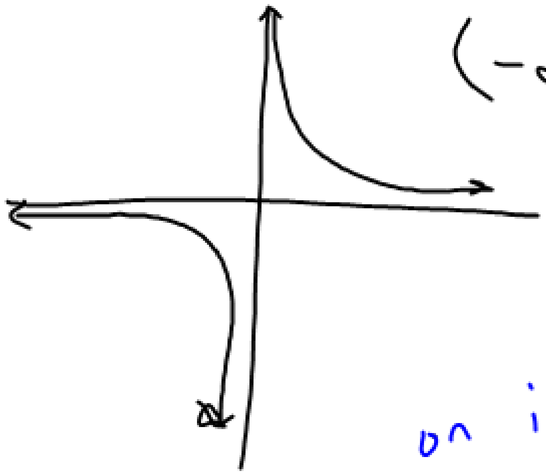


Continuous function → continuous at every on its domain

yes All $\mathbb{R}$ $x \neq 0$

continuous on an interval – continuous at every pt on the interval

$f(x) = \frac{1}{x}$

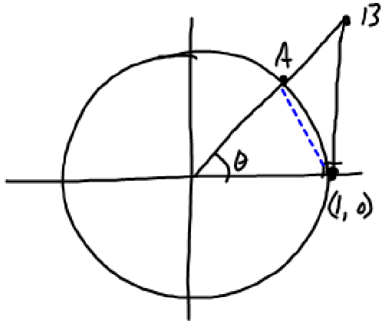


Domain

$(-\infty, 0) \cup (0, \infty)$

on interval $[-1, 1]$ → Not continuous infinite discontinuity

$\lim_{x \to 0} \frac{\sin x}{x}$ – Visualize a sector squeezed between 2 triangles



$A(\cos\theta, \sin\theta)$

$B(1, \tan\theta)$

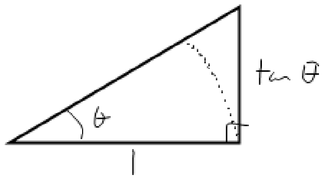
The sector is less than one △ bigger than the other

$$\frac{\sin\theta}{2} \le \frac{\theta}{2} \le \frac{\tan\theta}{2}$$

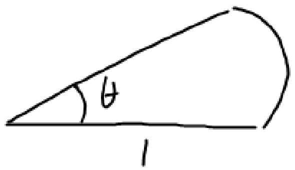
mult by $\frac{2}{\sin\theta}$

$$1 \le \frac{\theta}{\sin\theta} \le \frac{1}{\cos\theta}$$

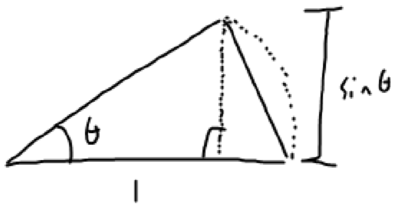
Area



$\frac{\tan\theta}{2}$



$\frac{\theta}{2}$ $\left[\frac{\theta}{2\pi}(\pi r^2) = \frac{r^2\theta}{2}\right]$ area sector



$\frac{\sin\theta}{2}$

Take reciprocal, switch inequalities

$$1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta$$

Use Sandwich Theorem and take limit

$$\lim_{x \to 0}\left(1 \ge \frac{\sin\theta}{\theta} \ge \cos\theta\right)$$

$$\lim_{x \to 0} 1 \ge \lim_{x \to 0} \frac{\sin\theta}{\theta} \ge \lim_{x \to 0} \cos\theta$$

$$\downarrow$$

$$1 \ge \lim_{x \to 0} \frac{\sin\theta}{\theta} \ge 1$$

$$\boxed{\text{So } \lim_{x \to 0} \frac{\sin\theta}{\theta} = 1}$$

P. 81 Exploration 1

Sect. 2.3 Q.R. #3
P.S. 1-9 (odd), 11-16, 18-51(3), 52, 53, 58